**8.812 Graduate Experimental Laboratory**
**MIT Department of Physics**
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### Exp. #1 Compton scattering

The objective of the experiment is two fold:

1. Demonstrate the particle feature of photons
2. Verify the Klein-Nishina calculation.

During this experiment you will:

- Study Compton kinematics and experimentally verify angle/energy relations.
- Derive from Feynman graphs the Klein Nishina cross section and measure it.
- Learn coincidence techniques, estimate energy resolution, determine accidentals.

This will be accomplished by observing the scattering of 661.6 KeV photons by electrons and measuring the energies of the scattered gamma rays as well as the energies of the recoil electrons.

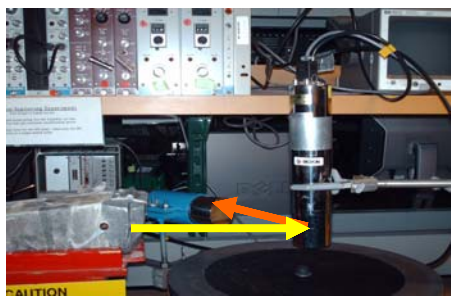

Fig.1 Experimental set up for Compton in 4- 359

### Preparation

1. Derive an expression for the final energy of a photon of initial energy E scattered from a free electron, initially at rest, at an angle θ from the forward direction, according to the Compton kinematics (see [1]). Describe four distinct ways in which 2 MeV photons can interact with matter.

2. Draw Feynman graphs for Compton scattering. Use Feynman rules to get the matrix elements (see Halzen & Martin, p.143).

3. Derive the Klein-Nishina differential and total cross sections.

4. Plot the expected energy spectrum of the scattered photons from a 662 keV primary beam for 20°, 45°, and 90° in the lab. For each angle tabulate the expected rates into a 20 $cm^2$ round counter 25 cm away from the target.

5. Sketch and explain the principle features of the pulse-height spectrum obtained from a NaI scintillation counter irradiated with 661.6 keV photons (see [2]). Give a qualitative reason for the scatter of energy values measured with these counters while observing the 'line'. The pulse height spectrum from 661.2 keV gamma rays striking the counter will show a line and a continuum. Explain both features with a sketch and give values in keV for the line and for the Compton edge.

**Physics History**

The successes of the quantum theory of blackbody spectra (Planck, 1901), the photoelectric effect (Einstein, 1905) and the hydrogen spectrum (Bohr, 1913) had established the idea that interactions between electromagnetic radiation of frequency *v* and matter occur through the emission or absorption of discrete quanta of energy E = *hv.* The next crucial step in the development of the modern concept of the photon as the particle of electromagnetic radiation was taken by Arthur Compton in the interpretation of experiments he initiated in 1920 to measure with precision the wavelengths of X-rays scattered from electrons in materials of low atomic number [1, 3]. He arrived at the formula for the "Compton shift" in the wavelength of incoherently scattered X-rays, namely (see [2], Appendix A)

$$\Delta\lambda = \frac{h}{mc}(1-\cos\theta) \qquad (1)$$

where *m* is the mass of the electron, and θ is the angle between the trajectories of the incident and scattered photon.

The agreement between Compton's experimental results confronted physicists and philosophers with the conceptual dilemma of the particle-wave duality of electromagnetic radiation. In particular, how was one to understand how each particle of light in a Young interference experiment goes simultaneously through two slits?

The dilemma was further compounded when, in 1924, DeBroglie put forward the idea that material particles (i.e. electrons, protons, atoms, etc.) should exhibit wave-like properties characterized by a wavelength λ related to their momentum *p* by the same formula as that for light quanta, namely $\lambda = h/p$. In 1927 the wave-like properties of electrons were discovered by Davisson and Germer in experiments on the reflection of electrons from crystals. A particularly interesting account of these developments is given by Compton and Allison in their classic treatise [5].

Fig.2 Compton (left)

## Measurements

1. Energy resolution of NaI counters.
   The total amount of scintillation light produced is closely proportional to the total amount of energy dissipated in the scattering processes. As a consequence the pulse height spectrum of a NaI scintillation counter exposed to monoenergetic gamma rays shows a distinct "photoelectric" peak which facilitates accurate calibration in terms of pulse height versus energy deposited. The counters can then be used to measure the energies of scattered photons and recoil electrons.

2. The energies of Compton-scattered gamma-ray photons and recoil electrons.
   These results will be compared with the predictions of Compton kinematics.

3. The frequency of occurrence as a function of angle $\theta$.
   These results will be compared with the predictions of the classical (Thompson) hypothesis of X-ray scattering and with the Klein-Nishina formula derived from relativistic quantum theory (QED).

4. The total cross section of electrons for Compton scattering.

**These are the results to be aimed for and included in the oral presentation and paper.**

## Schedule

The net measurements can be finished in 10-15 hours (large angle measurements can be run overnight) with proper completion of the preparation questions before starting. The solutions are mandatory and will be graded. The following is a suggestion:

Day 1 Familiarize with the hardware setup and the control programs.
Day 2 Study linearity of the energy response and tune coincidence measurements with $Na^{22}$.
Day 3 Verify calibration and "0" angle. Start coincidence measurements at "small" angles. Set to a "large" angle for over night.
Day 4 Continue measurements for different angles. Evaluate during longer runs.
Day 5 Measure the total cross section and repeat suspicious data.

## Experimental Setup

The experimental setup employs a radioactive source of 661.6 keV photons from $^{137}Cs$, and two NaI scintillation counters. One counter serves as the scattering target and measures the energy of the recoil electrons; the other counter detects the scattered photons and measures the energy they deposit. Both the target and scatter counters have a scintillator consisting of a 2”x2” cylinder of thallium-activated sodium iodide optically coupled to a photomultiplier. A 661.6 keV photon traversing sodium iodide has about equal probability of undergoing photoelectric absorption and Compton scattering.

The experimental arrangement for the Compton experiment is shown schematically in Figure 3. The 2”x2” cylindrical “recoil electron” or “target” scintillator detector (Canberra Model 802-3/2007) is irradiated by a beam of 661.6 keV photons emitted by ≈ 100 µcuries (≈ 1 µgram!) of $^{137}Cs$ located at the end of a hole in a large lead brick which acts as a gamma-ray “howitzer”.

If a photon entering the target scintillator scatters from a loosely bound (effectively free) electron, the resulting recoil electron may lose all of its energy in the target, causing a scintillation pulse with an amplitude proportional to the energy of the recoil electron. If the scattered photon emerges from the target scintillator without further interaction, hits the NaI crystal of the “scattered photon” detector, it will have a substantial probability of depositing all of its energy by a single photoelectric interaction or by a sequence of Compton scatterings and photoelectric interactions. This will produce a scintillation pulse that contributes to the “photopeak” of the pulse height spectrum. The median channel of the photopeak in the multichannel analyzer (MCA) display is a good measure of the median energy of the detected photons. If the scattered photon undergoes a Compton scattering in the scatter-scintillator and then escapes, the resulting pulse, with a size proportional to the energy of the recoil electron, will be registered in the *Compton recoil continuum* of the spectrum. Experimental details are considered in [7, 8, 9, 9, 10, 11].

**Coincidence:** A true Compton event will produce almost coincident signals in both detectors. If the logic (square) discriminator outputs overlap, the coincidence (‘AND’) produces an output ‘YES’ signal to the gate generator, which in turn notifies the MCA in the computer to analyze the pulse heights generated by this event. Most likely it is a ‘Compton event’; however there is a chance that two different (background) events accidentally occurred within the overlap time $2\tau$ given by the discriminator outputs. From a scatter counter with rate $n_s$ and target counter with rate $n_t$ the accidental rate

$$n_{acc} = 2\tau \; n_s n_t \qquad (2)$$

is small unless you have ‘noise’ increasing $n_s$ or $n_t$. A good method to set up the coincidence uses a $Na^{22}$ source placed in between the two detectors. WHY? But for Compton coincidences consider that for small $\theta$ the recoil (target) energy and hence signal are small. If the signals are below your discriminator threshold you fail to see coincident Compton events at small $\theta$.

**Gating (tricky):** In principle, the gate should tell the MCA ‘ahead of time’ that a good event is coming. With the present electronics, this is not possible so we try to catch the majority of the signal (Fig. 4) including the peak. The analog pulse shown is derived from the ‘unipolar’ output of the amplifier.

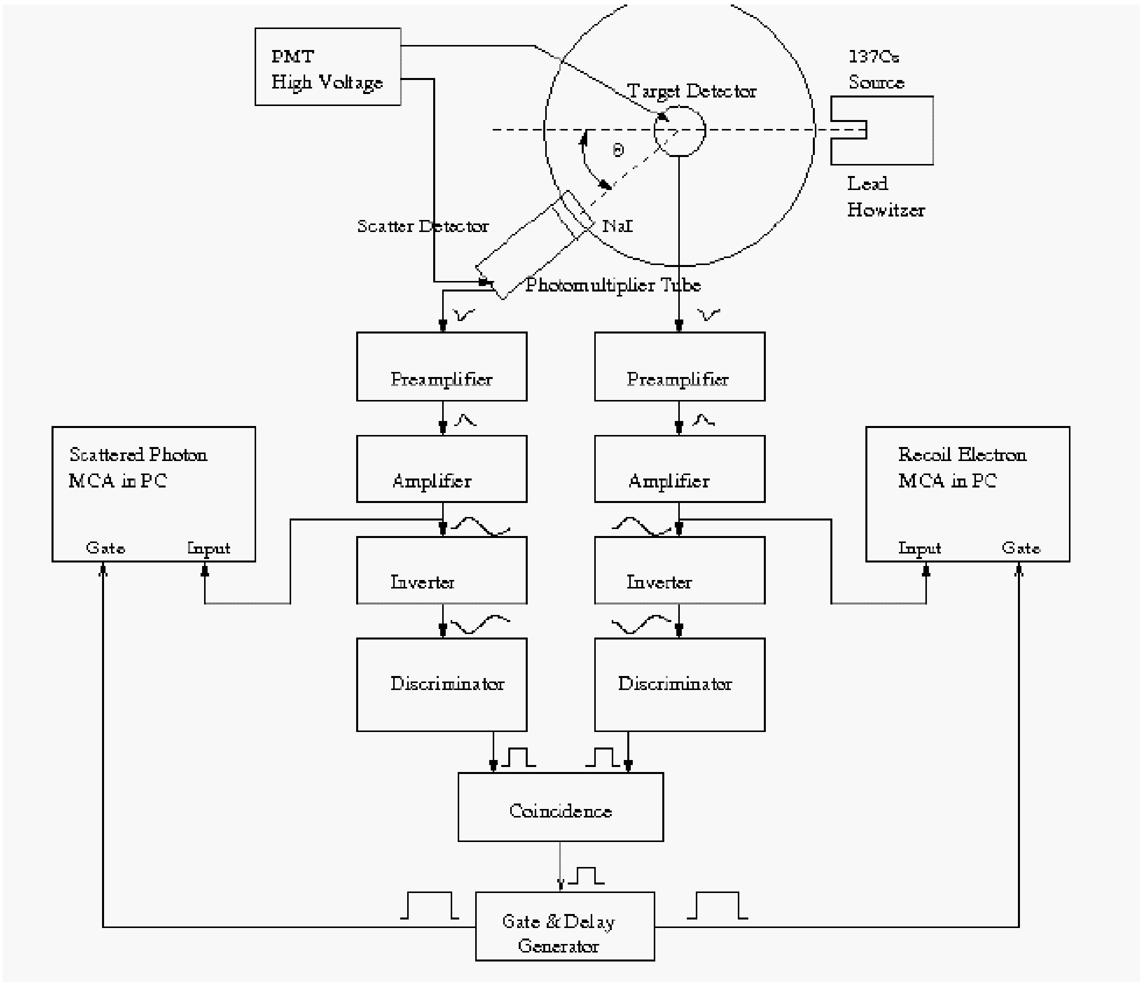


FIG. 3: Schematic diagram for Compton Scattering. Most of the equipment (NIM Crate, NIM modules and detectors) are on the lab bench; the two MCA cards are installed inside of the computer.

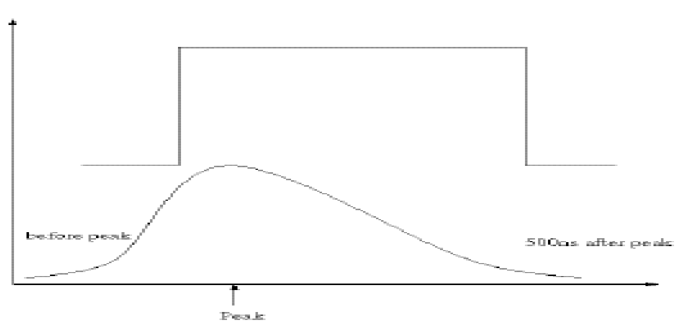


FIG. 4: Time relation between the photomultiplier tube signal (amplified and inverted) whose pulse height is to be measured, and the gate pulse from the Gate- Delay Generator NIM module. The gate pulse needs to arrive before the peak of the pulse and persist for ≥ 0.5μsec after.

## EXPERIMENT

Keep the lead door of the gamma-ray howitzer closed when not in use.

Calibration: Set up and test the experimental arrangement with 511 keV annihilation photons from a $^{22}$Na calibration source. $^{22}$Na, a radioactive isotope of sodium, decays to an excited state of $^{22}$Ne by emission of an anti-electron, i.e. a positron. The excited neon nucleus, in turn, decays quickly to its ground state by emission of a 1.27 MeV photon. Meanwhile, the positron comes to rest in the material in which the sodium isotope is embedded, combines with an electron to form an electrically neutral electronic atom called positronium (discovered at MIT in 1947 by Prof. Martin Deutsch). The latter lives about $10^{-7}$ s until the electron and positron annihilate yielding two 511 keV (= $m_e c^2$) photons traveling in opposite directions. Thus $^{22}$Na is a handy source of pairs of monoenergetic photons traveling in opposite directions which can be used to test a coincidence detector system.

a) Turn on the photoamplifier (+1000V), put the $Na^{22}$ source close to the Target or Scatter counter
b) Watch signals at the amplifier output, adjust the gain to about +7V
c) Record each peak by MCA from $^{137}$Cs (661.6 KeV), $^{22}$Na (511 KeV), and $^{133}$Ba (356, 302 and 81 KeV plus other lines) each of which produces easily identified photopeaks. Be aware that too high a counting rate will spoil the spectrum by overlapping signals and amplifier saturation.
d) Evaluate i) linearity, and ii) resolution.

Test the coincidence logic. Place the $^{22}$Na source between the two scintillators so that pairs of annihilation photons traveling in precisely opposite directions can produce coincident pulses in the two counters. Compare the spectrum with and without the coincidence requirement, and observe the effect of moving the source in and out of the straight line between the scintillators in order to observe hits by a pair of annihilation photons.

### Compton Scattering – Measurement at different angles

Use $^{137}$Cs to measure the energies of the scattered gamma rays and the recoil electrons as functions of the scattering angle $\theta$.

Measure the "profile" of the beam and determine $\theta = 0$. Check that the beam profile is symmetric about a line from the source through the center of the turntable to insure that the target counter will be uniformly illuminated when it is placed at the center.

Adjust the discriminators to insure that they will generate coincident pulses whenever a Compton scattering occurs in the target scintillator with a scattering angle at 20°. For a preliminary test, connect the (inverted) output of the target amplifier to the inputs of both discriminators. Simultaneously connect the output of the target amplifier to the "INPUT" BNC connector on the MCA card in the PC. Connect the output of the coincidence circuit to the gate generator to the "GATE" BNC on the MCA card. Using the Maestro-32 software package set the discriminators as low as possible without getting into the rapidly rising spectrum of photomultiplier noise.

**Measure the median energy $E'_\gamma$ of the scattered photons and the median Energy $E_e$ of recoil electrons as functions of the position angle $\theta'$ of the scattered photon detector.**

With both the source in the howitzer and the scintillator of the scattered photon detector located ≈ 40 cm from the recoil electron detector, set the position angle of the scatter counter at 90°. With the coincidence requirement enabled, accumulate a spectrum of scattered photon detector pulses on the 'scatter' MCA and a spectrum of recoil electron detector pulses on the 'target' MCA and measure the median channel of the pulses in the photoelectric peaks. Repeat this dual measurement at several position angles from just outside the primary beam profile to as near to 180° as you can get. A good strategy is to take data at widely separated position angles, say 90°, 30°, 120°, 150° and then fill in when you have time. Plot the raw results (median energy against position angle) as you go along to guide your measurement strategy. It is wise to check the calibration between each measurement by switching the (software based) GATE setting to 'OFF' and recording in your notebook the median channels of the photopeaks produced by 661.6 KeV photons from the $^{137}$Cs calibration source, the 511 KeV photons from $^{22}$Na, and the 356 KeV and 81 KeV photons of $^{133}$Ba.

*Record the integration time of each measurement to find the rate as a function of $\theta$. You need it later to compare with predictions***.**

**Total Scattering Cross Section**

The plastic scintillator blocks used as absorbers in this measurement are made from polyvinyltoluene ($C_{10}H_{11}$, density = 1.032 g/cc, www.bicron.com) composed of almost equal numbers of carbon and hydrogen atoms. The most tightly bound electrons are in the K-shells of the carbon atoms where they are bound by 0.277 KeV, which is small compared to 662 KeV. In fact, many other materials can be used providing approximately 'free' electrons. This material resembles most modern plastic scintillators.

Place the scatter counter at 0° and remove the target counter. Measure the counting rates in the 661.6 KeV photopeak of the NaI scatter counter with no absorber and with three or more different thicknesses of plastic scintillator placed just in front of the exit hole of the howitzer. WHY not in front of the detector? Determine the thicknesses (in g·cm$^{-2}$) of the scintillator blocks by measuring their dimensions and mass. Plot the measured rate as a function of thickness and check the validity of your data expected for exponential attenuation.

## Guide to analysis

1. Plot $E_{target} + E_{scatter}$ vs. $\theta$. Does it add up to 661.6 KeV? In each case specify/mark the errors.
2. Estimate the solid angle $\Delta\Omega$ suspended by the counters for later $d\sigma/d\Omega$ evaluation.
3. Verify the kinematics by comparing your angular measurements of energies from the scattered photons and the recoil electrons with the Compton formula and the classical Thomson expectation.
4. Display the measured 'rate' as a function of $\theta$ and compare $d\sigma/d\Omega$ (measured) to the Klein-Nishina relation you calculated.
5. Evaluate the total cross section per electron. The attenuation of a collimated beam of particles by interactions in a slab of material of thickness x (cm) is $\mathbf{I}(x) = I_0 e^{-\mu x}$ where $\mu$ is the total linear attenuation coefficient ($cm^{-1}$). In plastic scintillator the attenuation of photons is due almost entirely to Compton scattering. Then $\sigma_{total} = \frac{\mu}{n_e}$ with $n_e$ the number of electrons/$cm^3$ in scintillator $(CH)_n$ material. Find $\mu$ from your data, keeping track of errors and obtain $\sigma_{total}$. Compare to the non QM Thomson calculation and to the theoretical result of Klein Nishina [12].

## Your paper

Try to stay within 8 pages figures included. Latex recommended, but not required. Close to PRL format. We recommend the classical sequence:

- WHY this experiment (Physics)
- WHAT was done (this paper's experimental approach)
- WHICH accuracy was achieved (errors)
- WHAT was learned?

In other words, a WWW(W) approach.

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## Appendices

### A: SCINTILLATION COUNTER EFFICIENCY ESTIMATE

As in all cross section measurements, the necessarily finites sizes of source, target and detector introduce into the analysis geometrical factors (acceptance) that require laborious multidimensional integrations for their evaluations. In our experiment we have a divergent beam of photons from a radioactive source of finite size that can interact at various depths in the target within certain ranges of solid angle to produce scattered photons that traverse the detector resulting in a convolution of the differential cross section with geometrical and attenuation factors represented by a multiple integral over at least ten variables with hideous limits of integration.

For a simple "Monte Carlo" calculation we will consider only single scattering events. We call $R$ and $H$ the radius and length, respectively, of the cylindrical scintillators which are the sensitive elements of the target and scatter counters. We assume that the source is so far from the target scintillator that the incident beam may be considered uniform and parallel (i.e. unidirectional). We assume that the distance $D$ of the scatter counter from the target is so large that the detected portion of the scattered radiation may be considered to comprise a parallel (but not uniform) beam.

Consider the scatterings that occur within a solid element of volume $rd\phi drdz$ at a position in the target scintillator with cylindrical coordinates $r, \phi, z$. The distance $x$ of this element from the point of entry of the incident photons into the target scintillator is given by the expression

$$x = R\{[1 - (q\sin\phi)^2]^{\frac{1}{2}} - q\cos\phi\} \qquad \text{(A1)}$$

where $q = r/R$. Similarly, for photons scattered from the point $P$ at the angle $\theta$ the distance $y$ of the element from the point of exit is

$$y = R\{[1 - (q\sin\psi)^2]^{\frac{1}{2}} - q\cos\psi\} \qquad \text{(A2)}$$

where $\psi = \pi - \theta + \varphi$.

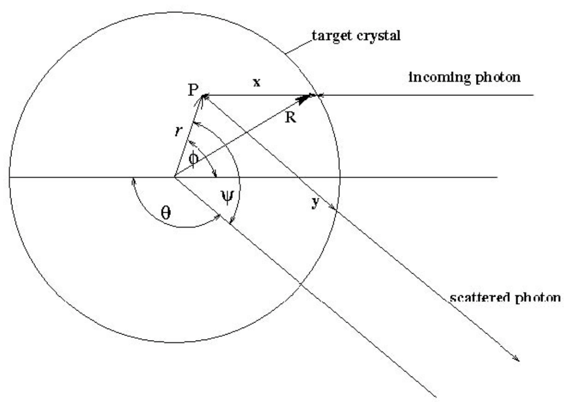


*FIG.4: Compton scattering geometry for acceptance estimate.*

We call α the total linear attenuation coefficient (units of $cm^{-1}$) of the target plastic scintillator for the incident photons, $\beta(\theta)$ the total linear attenuation coefficient of the plastic for photons scattered at angle θ and $\gamma(\theta)$ the total linear attenuation coefficient of the scatter counter NaI scintillator for those scattered photons. The efficiency for detection of scattering events that occur at a given position in the target scintillator is $\exp(-\beta y)[1-\exp(-\gamma H)]$ which represents the probability that the scattered photon will escape from the target counter multiplied by the probability that the scattered photon interacts in the scatter counter.

We must now find the average of this efficiency over the target scintillator. Recalling the assumption that the incident beam is parallel and uniform, the weighting factor is $I_o \exp(-\alpha x)$ which is the intensity of the incident beam at the point of interaction inside the target scintillator. We can now write for the average efficiency the expression

$$\eta(\theta) = \frac{\int \exp(-\alpha x)\exp(-\beta y)[1-\exp(-\gamma H)]dV}{\int \exp(-\alpha x)dV} \quad \text{(A3)}$$

where the integrals are computed over the volume of the target scintillator. We note that $\eta \rightarrow 1$ as $\alpha \rightarrow 0, \beta \rightarrow 0$.

The total attenuation coefficients are functions of the energy of the scattered photon and the energy of the scattered photon is a function of the scattering angle. According to the Compton kinematics:

$$E\gamma(\theta) = \frac{E_0}{1+(E_0/mc^2)(1-\cos\theta)} \quad \text{(A4)}$$

In the energy range from 0.2 to 0.7 MeV the total attenuation coefficient in sodium iodide can be fairly well represented by the formula

$$\mu = 0.514\left(\frac{E}{100keV}\right)^{-0.368} + 5.51\left(\frac{E}{100keV}\right)^{-2.78} cm^{-1} \quad \text{(A5)}$$

and in plastic by the formula

$$\mu = 0.177\left(\frac{E}{100keV}\right)^{-0.37} cm^{-1} \quad \text{(A6)}$$

The quantity $\eta$ can be evaluated as a function of $\theta$ by numerical integration.

## B: DERIVATION OF THE COMPTON SCATTERING FORMULA

Consider the interaction between a photon with momentum vector $\mathbf{P}\gamma$ (and zero rest mass) and an electron initially at rest with rest mass $m_e$. Call $\mathbf{P}'_\gamma$ and $\mathbf{P}'_e$ the momenta of the photon and the electron after the interaction, respectively. By conservation of momentum, the relation between the initial and final momenta is

$$\mathbf{P}_\gamma = \mathbf{P}'_\gamma + \mathbf{P}'_e \quad \text{(B1)}$$

Rearranging and squaring both sides we obtain

$$p^2_\gamma + p'^2_\gamma - 2\mathbf{P}_\gamma \cdot \mathbf{P}'_\gamma = \mathbf{P}'^2_e \quad \text{(B2)}$$

The total relativistic energy E and momentum P of a particle are related to its rest mass m by the invariant relation $\mathbf{P} \cdot \mathbf{P}\, c^2 - \mathbf{E}^2 = - m^2c^4$. By conservation of energy,

$$p_\gamma + m_e c = p'_\gamma + \sqrt{m_e^2 c^2 + p'^2_e} \quad \text{(B3)}$$

Rearranging and squaring both sides

$$p^2_\gamma + p'^2_\gamma - 2p_\gamma p'_\gamma + 2m_e c(p_\gamma - p'_\gamma) + m^2_e c^2 = m^2_e c^2 + p'^2_e \quad \text{(B4)}$$

Subtraction of (B2) from (B4) and rearrangement yields

$$m_e c(p_\gamma - p'_\gamma) = p_\gamma p'_\gamma - \mathbf{P}_\gamma \, . \, \mathbf{P}'_\gamma \quad \text{(B5)}$$

Finally, dividing both sides by $m_e c p_\gamma p'_\gamma$, we obtain for the relation between the energies of the incident and the scattered photons the equation

$$\frac{1}{E'_\gamma} - \frac{1}{E_\gamma} = \frac{1}{\mathrm{m_e} c^2}(1-\cos\theta) \quad \text{(B6)}$$

Where $\theta$ is the angle between their final momentum vectors.